Finite Population Correction Factor

If we know the population size — of course, we often do not — but if we do, then we can use that information to reduce our sampling error (ε). What we do is multiply ε by the finite population correction factor, whose formula follows.

$$\sqrt{\frac{N-n}{N-1}}$$

So, let's say that we have the following data, which I have modified from p. 283 of our textbook:

$$\begin{aligned} \bar{x} &= \$1\,076.39 \\ s &= \$273.62 \\ n &= 77 \\ N &= 1\,000 \end{aligned}$$

Understand that *N* (the population size) is the value that we don't usually have.

Let's figure out a 95% confidence interval without making use of *N*:

$$\begin{aligned} &\left(\bar{x} - t_{\alpha/2,df}\frac{s}{\sqrt{n}},\quad \bar{x} + t_{\alpha/2,df}\frac{s}{\sqrt{n}}\right) \\ &= \left(\$1\,076.39 - 1.9917\frac{\$273.62}{\sqrt{77}},\quad \$1\,076.39 + 1.9917\frac{\$273.62}{\sqrt{77}}\right) \\ &= (\$1\,014.29,\quad \$1138.49) \end{aligned}$$

[And you will get a very slightly different, and more correct, answer if you do the interval using your calculator function.]

Now, let's multiply ε by the finite population correction factor, as such:

$$\begin{aligned} &\left(\bar{x} - t_{\alpha/2,df}\frac{s}{\sqrt{n}}\sqrt{\frac{N-n}{N-1}},\quad \bar{x} + t_{\alpha/2,df}\frac{s}{\sqrt{n}}\sqrt{\frac{N-n}{N-1}}\right) \\ &= \left(\$1\,076.39 - 1.9917\frac{\$273.62}{\sqrt{77}}\sqrt{\frac{1\,000-77}{1\,000-1}},\quad \$1,076.39 + 1.9917\frac{\$273.62}{\sqrt{77}}\sqrt{\frac{1\,000-77}{1\,000-1}}\right) \\ &= (\$1\,016.69,\quad \$1136.09) \end{aligned}$$

What have we gained? A more precise confidence interval, without any loss of confidence.

And how did we accomplish this? By reducing the sampling error some 3.9%. Notice the value of the finite population correction factor in the present case:

$$\sqrt{\frac{N-n}{N-1}} = \sqrt{\frac{1\,000-77}{1\,000-1}} \doteq 96.1\%$$

