Math 115 Reference Sheet

Exponential Expressions

$(ab)^n = a^n b^n$ $\qquad (\frac{a}{b})^n = \frac{a^n}{b^n}$ $\qquad (b^m)^n = b^{mn}$

$b^m b^n = b^{m+n}$ $\qquad \frac{b^m}{b^n} = b^{m-n}$ $\qquad b^{-n} = \frac{1}{b^n}$ $\qquad \frac{1}{b^{-n}} = b^n$

$a^{\frac{m}{n}} = (\sqrt[n]{a^m})$ $\qquad a^{\frac{m}{n}} = (\sqrt[n]{a})^m$ $\qquad a^{-\frac{m}{n}} = \frac{1}{a^{\frac{m}{n}}}$ $\qquad a^{\frac{1}{n}} = \sqrt[n]{a}$

Special Factorizations

$$A^2 - B^2 = (A+B)(A-B)$$

$A^2 + 2AB + B^2 = (A+B)^2$ $\qquad A^2 - 2AB + B^2 = (A-B)^2$

$A^3 + B^3 = (A+B)(A^2 - AB + B^2)$ $\qquad A^3 - B^3 = (A-B)(A^2 + AB + B^2)$

Linear Functions and Slope

Slope: $m = \frac{y_2 - y_1}{x_2 - x_1}$ $\qquad$ **Point − Slope Form:** $y - y_1 = m(x - x_1)$

Slope − Intercept Form: $y = mx + b$ $\qquad$ **General Form:** $Ax + By + C = 0$

Quadratic Equations and Functions

If $ax^2 + bx + c = 0$ **then** $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

Standard form of a parabola: $f(x) = a(x-h)^2 + k, \ a \neq 0$; **The vertex is** (h, k)

If $f(x) = ax^2 + bx + c$ **then** $h = -\frac{b}{2a}$ **and** $k = f\left(-\frac{b}{2a}\right) = c - \frac{b^2}{4a}$

Functions, Theorems, and Concepts

Leading Coefficient Test for n^{th} degree polynomial functions:

For n odd and $a_n > 0$, graph falls to the left and rises to the right
For n odd and $a_n < 0$, graph rises to the left and falls to the right
For n even and $a_n > 0$, graph rises to the left and rises to the right
For n even and $a_n < 0$, graph falls to the left and falls to the right

***x*-intercepts** occur where $f(x) = 0$; ***y*-intercepts** occur, where $x = 0$

Even Functions: $f(-x) = f(x)$ symmetric with respect to the *y*-axis
Odd Functions: $f(-x) = -f(x)$ symmetric with respect to the origin

The Remainder Theorem: If the polynomial $f(x)$ is divided by $(x-c)$, then the remainder is $f(c)$.
The Factor Theorem: Let $f(x)$ be the polynomial.
If $f(c) = 0$, then $x-c$ is a factor of $f(x)$. If $x-c$ is a factor of $f(x)$, then $f(c) = 0$

The Rational Zero Theorem: If $f(x) = a_n x^n + a_{n-1}x^{n-1} + \cdots + a_1 x + a_0$ has *integer* coefficients and $\frac{p}{q}$ (where $\frac{p}{q}$ is reduced to lowest terms) is a rational zero of f, then p is a factor of the constant term a_0, and q is a factor of the leading coefficient a_n.
Possible Rational Zeros = $\frac{Factors\ of\ the\ constant\ term}{Factors\ of\ the\ leading\ coefficient}$
Locating Horizontal Asymptotes:
Let f be the rational function given by $\frac{a_n x^n + a_{n-1}x^{n-1} + \cdots + a_1 x + a_0}{b_m x^m + b_{m-1}x^{m-1} + \cdots + b_1 x + b_0}$, $a_n \neq 0$, $b_n \neq 0$
The degree of the numerator is *n*. The degree of the denominator is *m*.

1. If $n < m$, the *x*-axis, or $y = 0$, is the horizontal asymptote of the graph of *f*.
2. If $n = m$, the line $y = \frac{a_n}{b_m}$ is the horizontal asymptote of the graph of *f*.
3. If $n > m$, the graph of *f* has no horizontal asymptote.

Exponentials and Logarithms

Compound Interest $A = P\left(1 + \frac{r}{n}\right)^{nt}$ **Continuous compounding** $A = Pe^{rt}$

$y = a^x$ **if and only if** $\log_a y = x$

$\log_a MN = \log_a M + \log_a N$ $\log_a \frac{M}{N} = \log_a M - \log_a N$ $\log_a M^r = r \log_a M$

$\log_a a^x = x$ $a^{\log_a x} = x$ $e^{\ln x} = x$ $\ln e^x = x$

$\ln e = 1$ $\log_a 1 = 0$ $\log_b M = \frac{\log M}{\log b}$ $\log_b M = \frac{\ln M}{\ln b}$