Gradians, Not Degrees:
How to Measure Navigational Velocity Using the Metric System

by
Stanley M. Max
Lecturer, Department of Mathematics, Towson University
Certified Metrication Specialist, U.S. Metric Association

Abstract

Even in countries where the metric system has been long established, knots are used to measure navigational velocity on the sea and in the air. This is surprising because a knot, which equals one nautical mile per hour, is not a metric unit. However, the knot is based upon the size and shape of the Earth, so this unit has natural appeal to navigators. Does a method exist to use the metric unit of kilometers per hour for measuring navigational velocity that is also based upon the size and shape of the Earth, and which would therefore provide equal appeal to navigators? Yes it does, provided that angular measurement is performed using gradians (also called gons) instead of degrees.

Measuring navigational velocity

In air and sea navigation throughout much of the world, horizontal velocity is measured in knots (kn). This is not a metric unit, which is to say that it does not comply with the international standards officially known as the International System of Units (known in short as SI). For a knot, the metric equivalent is kilometers per hour (km/h). I strongly advocate for the metric system, and I decry the continued and stubborn refusal of the United States to fully metricate its system of weights and measures. I believe that having to learn two systems of measurement — one of which (the inch-pound system) is archaic and inefficient — very much contributes to the poor mathematical skills exhibited by Americans.

The fact that non-metric units are used in U.S. navigation causes me no surprise. However, I am rather astonished to learn that these units are utilized even in much of the rest of the world, where the metric system has long taken root. Why would countries that otherwise conduct their affairs with wall-to-wall metric units not also put them into place when it comes to navigation?

Here is why the knot is used to measure velocity. It relates directly to the mostly spherical shape of the Earth.

Defining a knot and a nautical mile

What is a knot, and why does it continue to hold sway? A knot is a unit of velocity equal to one nautical mile per hour. So what is a nautical mile? A nautical mile equals one minute of arc on the surface of the Earth.

Imagine a point on the equator, and we want to measure the distance from that point to the North Pole. In angular measurement that distance equals 90 degrees, and since each degree contains 60 minutes, the distance from the equator to the North Pole amounts to 5400 minutes. The distance intercepted (or subtended) by each one of those arcminutes is called a nautical mile. Thus, in angular measurement the distance from the equator to the North Pole equal 5400 minutes, and in linear measurement that same distance is 5400 nautical miles. A nautical mile, which measures approximately 6076 feet, is the arc length of one arcminute, whereas a statute mile, which measures 5280 feet, is the chord length of one arcminute.

The natural relationship of a nautical mile to the size and shape of the Earth makes this unit of measurement appealing to navigators. For hundreds of years — well before the modern age — sailors have been able to measure latitude. Let us make sure we grasp this. Experienced sailors 500 years ago and even longer than that knew how to determine how far north or south they were from somewhere else, and thus they would have clearly understood the meaning of a nautical mile. (Incidentally, measuring longitude — measuring how far east or west you are — is not easily accomplished, and mariners mastered that skill only within the last 250 years or so. I recommend the fascinating account by Dava Sobel in her book entitled *Longitude: The True Story of a Lone Genius Who Solved the Greatest Scientific Problem of His Time*.)

Identifying a great circle

A nautical mile is one minute of arc along any great circle of the Earth. A great circle of the Earth is the circle that would be formed on a plane if that plane sliced through the center of the Earth cutting the planet into two halves. The distance along the entire length of a great circle is the length of the circumference of the Earth.

Thus, any plane going through the Earth from the North Pole to the South Pole forms a great circle. For example, the prime meridian, which passes through Greenwich, England, lies on a great circle. But any of the infinitely many meridians also form great circles.

A great circle does not have to extend in a north-south direction. The equator forms a great circle. However, the equator is the only line of latitude that lies on a great circle. Any line of latitude to the north or the south of the equator does not form great circle because it does not pass through the Earth's center.

Finally, a great circle can also be formed obliquely. The only requirement is that the imaginary plane slicing through the Earth needs to go through the center of the planet. As long as that condition is met then the plane can lie non-orthogonally to the equator.

Gradians to the rescue

As a supporter of the metric system, therefore, am I out of luck? Since it measures velocity on the basis of the nautical mile, the knot seems to be the natural choice to use. Indeed, the knot's relationship to planet Earth accounts for its long-standing use right down to the present day, even in countries where the metric system is firmly entrenched. A nautical mile equals one minute of arc length along a great circle, and counting knots tells the navigator how long it takes to travel that distance.

Yes, the knot would be the best unit of measurement if the degree were the only way to measure angles. But it is not. Enter the gradian.

To be clear, I am not talking about the radian. In the modern metric system the radian is the SI unit for measuring plane angles, and the steradian is the SI unit for measuring solid angles. The gradian measures angles using a method similar to using degrees, but somewhat different and actually better. Historically, the gradian came into existence in France at the same time that the French revolutionary government established the metric system in the 1790s.

Before proceeding, some necessary terminology. In German and in Germanic Scandinavian languages, the word *grad* means *degree*, so using the word gradian led to confusion. To avoid it, a gradian is now called a gon, taken from the Greek word *gonia*, which means angle. I learned about the Greek origin of the word from the amazingly useful website *How Many? A Dictionary of Units of Measurement* by Russ Rowlett of the University of North Carolina at Chapel Hill (see http://www.unc.edu/~rowlett/units/index.html). As a mathematical symbol, the word gon should be placed after the number with a space in between, or the superscript g can be used.

As we know, a quarter distance around a circle equals 90°, a half distance equals 180°, a full turn is 360°, and so on. Using gons, a quarter distance around a circle equals 100 gon, a half distance equals 200 gon, a full turn is 400 gon, and so on. So each quadrant on the Cartesian plane measures 100 gon, a more convenient value than 90 when using degrees.

I will provide a few equations to show how angular measurement works with gons:

25 gon or $25^g = 22.5°$

50 gon or $50^g = 45°$

75 gon or $75^g = 67.5°$

100 gon or $100^g = 90°$

150 gon or $150^g = 135°$

200 gon or $200^g = 180°$

400 gon or $400^g = 360°$

Gons and kilometers

What do gons have to do with navigation? To answer that, let us understand why the meter has the length that it does. That length is not arbitrary, but rather it is based in nature. The French revolutionary government, upon establishing the metric system in the 1790s, commissioned a study of the length of the meridian passing from Dunkirk to Barcelona, and the government decreed that one ten-millionth of the arc extending from the North Pole to the Equator along this meridian would constitute a meter. Hence, a quarter circumference of the globe measures ten million meters, or 10 000 kilometers.

Thus, 10 000 kilometers measures one quarter of a great circle, and 40 000 kilometers is the circumference of the Earth using metric units. This relates linear measurement (kilometers) to angular measurement (gons). Here are a few equations demonstrating that relationship:

$$10\,000 \text{ km} = 100 \text{ gon}$$

$$100 \text{ km} = 1 \text{ gon}$$

$$1 \text{ km} = \text{cgon, where cgon} = \text{centigon}$$

Here are analogous equations using nautical miles and degrees:

$$5400 \text{ nautical miles} = 90°$$

$$60 \text{ nautical miles} = 1°$$

$$6076 \text{ feet} \approx 1 \text{ arcminute}$$

The metric way (kilometers and gons) versus the nonmetric way (nautical miles and degrees)

I have given due praise to Russ Rowlett's website on measurement units. However, when he makes this statement under the entry for the grad, he is wrong: "Although many calculators will display angle measurements in grads as well as degrees or radians, it is difficult to find actual applications of the grad today."

No. The grad (the gon) establishes the connection between kilometers — the metric unit of linear measurement — and angular measurement. Using gons, navigators can free themselves from having to use nonmetric units to draw the connection in the natural world between linear and angular measurement. Using gons, navigators do not have to remind themselves that a nautical mile equals one minute of arc length along a great circle of the Earth. Instead, they can make the same connection by reminding themselves that a kilometer equals one centigon of arc length along a great circle. So, by knowing how many kilometers per hour they are traveling, they simultaneously know how long it takes them to travel one centigon of arc length on the surface of the Earth. This is as natural a way to think about navigational distance as using the knot. And it is better, because they are using the metric system.