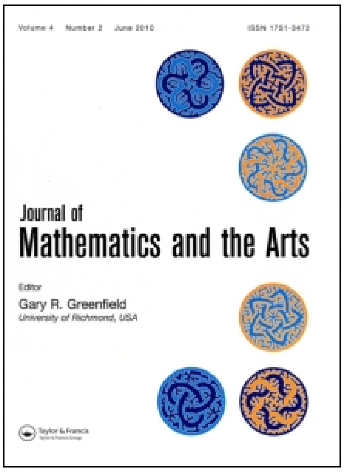

Journal of Mathematics and the Arts
Publication details, including instructions for authors and subscription information:
http://www.informaworld.com/smpp/title~content=t755420531

The 'Golden Canon' of book-page construction: proving the proportions geometrically

Stanley M. Max[a]
[a] Department of Mathematics and Statistics, University of Southern Maine, Portland, ME 04104, USA

Online publication date: 10 September 2010

To cite this Article Max, Stanley M.(2010) 'The 'Golden Canon' of book-page construction: proving the proportions geometrically', Journal of Mathematics and the Arts, 4: 3, 137 — 141
To link to this Article: DOI: 10.1080/17513470903458205
URL: http://dx.doi.org/10.1080/17513470903458205

Journal of Mathematics and the Arts
Vol. 4, No. 3, September 2010, 137–141

The 'Golden Canon' of book-page construction: proving the proportions geometrically

Stanley M. Max*

Department of Mathematics and Statistics, University of Southern Maine, Portland, ME 04104, USA

(*Received 24 November 2008; final version received 27 October 2009*)

Scholars of book arts have speculated upon and may have proven a deliberate method by which scribes and printers of past centuries placed the text area in manuscript and printed books on the page to render certain specific text-to-page ratios. According to this presumed method, the size and proportions of the text area would assume specific mathematical properties with respect to the size and proportions of the page. In 1953, one book-arts scholar, Jan Tschichold, referred to the 'Golden Canon of Gothic book page construction', and thus coined the term 'golden canon' (which, it should be noted, has nothing to do with the golden section). These scholars have stated that the golden canon results in these properties, but none of them have shown why it does. Using basic geometry, and in particular similar triangles, this article addresses the question.

Keywords: book arts; golden canon; page layout; similar triangles

AMS Subject Classifications: 01A35; 01A40; 51-03

1. Background

Several scholars of book arts have speculated upon and may have proven a deliberate method by which scribes and printers of past centuries placed the text area in manuscript and printed books on the page to render certain specific text-to-page ratios. Especially notable among these scholars was Jan Tschichold [6], an eminent student of book design and typography. 'After much toilsome work', Tschichold wrote in 1975, 'I finally succeeded, in 1953, in reconstructing the Golden Canon of book page construction as it was used during late Gothic times by the finest of scribes'. Tschichold – who had now coined the term 'golden canon' – went on to say that what he 'uncovered as the canon of the manuscript writers, Raúl Rosarivo [another book-arts scholar] proved to have been Gutenberg's canon as well'. In turn, Sam Somerville [5] synthesized the work of Tschichold, Rosarivo and other scholars in a noteworthy article published in 1983.

So then, what is the golden canon of book-page construction? It is a technique, Somerville explains, that places the text on a page with three predictable and consistent results. In my mind, the fact that the same text layout can always be reproduced yields both a certain aesthetic tranquillity as well as a scientific precision to the process of text composition. Shall we accept the claim of Rosarivo and Tschichold that medieval scribes and early modern printers were placing text on the page according to a certain set procedure that Tschichold called the golden canon? That remains an open question and one inviting further investigation. If we do accept the claim, however, then the fact that such results were achieved by the High Middle Ages makes the student of European cultural history, and especially the student of book arts, marvel at the mathematical sophistication already achieved by that time. I think that the golden canon of book-page construction evokes the building skill of the Gothic architects who designed and constructed the magnificent European cathedrals. What those architects achieved in constructing a building, so too did scribes and printers accomplish in putting text onto a page.

To get down to brass tacks, just what are those three predictable and consistent results? First, the text area has the same proportion of width to height as the page does. Second, the width of the outer margin of the text area ends up being exactly twice the width of the inner margin, and the width of the bottom margin becomes exactly twice the width of the top margin. Third, the ratios of inner-margin width to top-margin width, and outer-margin width to bottom-margin width, turn out to be the very same ratio of page width to page height. (Incidentally, the reader should not confuse the golden canon with the golden section, which of course is the famous mathematical construction that also involves ratios but that does not necessarily have anything to do with page layout.)

Tschichold's article showed how to construct a page using the golden canon so that it worked with

*Email: smax@usm.maine.edu

ISSN 1751–3472 print/ISSN 1751–3480 online

DOI: 10.1080/17513470903458205
http://www.informaworld.com

particular ratios of page width to page height, but he did not explain why it worked with any page ratio. In 1919, Jay Hambidge [2,3] showed designs having what he called 'dynamic symmetry', and Tschichold based his article upon those designs. However, Hambidge did not refer to specific text-to-page ratios. In 1932, Edward B. Edwards [1] demonstrated how to lay out the text on a page so that it yielded the proportions that Tschichold years later called the golden canon, but he did not address all three ratios emanating from the golden canon. Other authors have also described page layouts resembling the golden canon, but none of them have fully explained the mathematical proportions behind their layouts (see http://en.wikipedia.org/wiki/Canons_of_page_construction and [4,7]). Somerville's synthesis clearly spelled out these three ratios produced by the golden canon, which I have restated in the paragraph above. Once more, however, Somerville did not explain why those results occur.

My article will refer to Somerville's synopsis of the golden canon, and one by one I will explain why the canon always produces specific ratios no matter the size or proportions of the page. To do this, I will rely upon the geometrical concept of similar triangles.

This article grew out of a summer programme in book arts that I took at the University of Southern Maine. One of the instructors in that programme, Nancy Leavitt, a noted book artist and calligrapher, introduced me to the subject of the golden canon, and she gave me a copy of Somerville's article. That article ignited my fascination with this topic.

2. Similar triangles

Let us briefly review the concept of similar triangles. Consider Figure 1, in which I have depicted two triangles so that the smaller triangle on the left, ABC, is the same shape but half the linear size as the larger triangle on the right, $A'B'C'$. (The smaller triangle ABC also occupies one-quarter the area of $A'B'C'$, but that fact does not concern us in this article.) Thus, the two triangles are similar (which we write as $\Delta ABC \sim \Delta A'B'C'$). The two triangles are not congruent since they have different sizes. If they were congruent, we would write that $\Delta ABC \cong \Delta A'B'C'$.

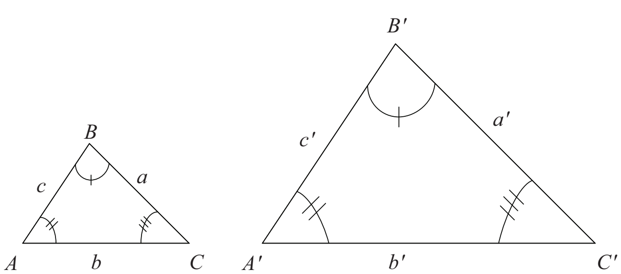

Figure 1. An example of two similar triangles. The smaller triangle on the left, ABC, is one-half the linear size of the larger triangle on the right, $A'B'C'$.

To explain necessary symbols, we observe that angle ABC has the same measurement as angle $A'B'C'$, which geometers write as $\measuredangle ABC \cong \measuredangle A'B'C'$. In the same manner, we have that $\measuredangle BAC \cong \measuredangle B'A'C'$ and that $\measuredangle ACB \cong \measuredangle A'C'B'$. As a result of these equalities of angle measurement, we have the situation that side AC corresponds to side $A'C'$ (geometers write that $AC \leftrightarrow A'C'$), side CB corresponds to side $C'B'$ ($CB \leftrightarrow C'B'$) and side BA corresponds to side $B'A'$ ($BA \leftrightarrow B'A'$).

So what consequence do these correspondences have? They mean that we have the following equal proportion:

$$\frac{AC}{A'C'} = \frac{CB}{C'B'} = \frac{BA}{B'A'}$$

Furthermore, since I have deliberately drawn the larger triangle to be twice the size of the smaller, so as to simplify the arithmetic of the ratios, we can quantify the ratios further. Using sides AC and $A'C'$ as an example (I could have used any one of the three sides), we have the following equations:

$$AC = \frac{1}{2}A'C' \text{ or } 2AC = A'C'$$

The ability to establish and quantify proportions has profound results for the golden canon, and indeed it provides the ticket to prove the golden-canon ratios by means of geometry.

3. Explanation of the diagrams

My article contains two figures besides the sample triangles in Figure 1. Figure 2, commonly depicted in books and articles on the subject, indicates how the text areas are placed on the verso (left-hand page) and

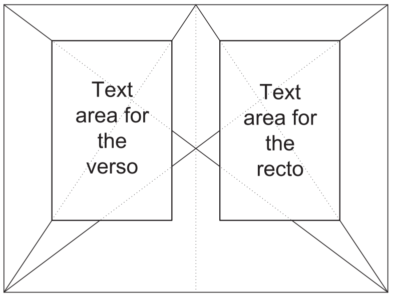

Figure 2. The placement of the text area on the verso and the recto according to the golden canon. The size of the text area was arbitrary – any size would fulfil the properties of the golden canon.

the recto (right-hand page) according to the golden canon. Somerville's 1983 article contains a similar illustration along with annotations and labels of key lines and angles. Observe that Figure 2 depicts a two-page layout with the dotted line separating the pages. Two diagonals cross the two-page spread each going to opposite corners. The verso also has a diagonal proceeding from the bottom-left to the top-right corner and the recto has its own diagonal going from the top-left to the bottom-right corner. The two smaller rectangles, one on each page, indicate a possible area for text, and I will shortly explain how to construct those rectangles. The text area that I chose was arbitrary – as long as the text block is constructed in the manner that I am about to describe, the ratios predicted by the golden canon always work.

In Figure 3, I have continued to utilize Somerville's labels to maintain consistency, but I have added numerous other labels to which I shall refer as I proceed with my argument. I have eliminated Somerville's annotation along with most of the lines from the verso so that I can concentrate on the recto. Everything that I establish regarding the recto immediately applies to the verso, and therefore we have no need to clutter the diagram.

In Figure 3, to which the remainder of this article applies, I show two main diagonals. One of them extends from the bottom left of the two-page layout to the top right and the other one goes from the top left of the two-page layout to the bottom right. I also show a diagonal on the verso extending from its bottom left to its top right, as well as a diagonal on the recto going from its top left to its bottom right. I also have a dotted line bisecting the two-page layout.

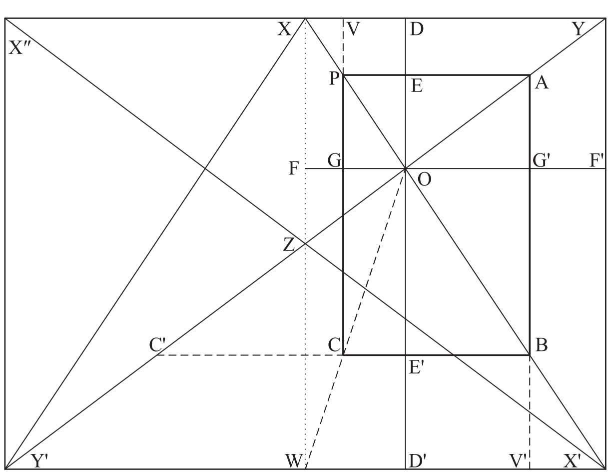

Figure 3. Geometrical development of the golden canon, concentrating on the recto. The same methodology would pertain to the verso.

We have that $\measuredangle X'XY \cong \measuredangle XX'Y'$, because they are congruent 'alternate interior angles'. A straight line intersecting parallel lines forms alternate interior angles. Here, of course, the parallel lines are the top and bottom edges of the page and the straight line is XX'. For the same reason, $\measuredangle X''YY' \cong \measuredangle X'Y'Y$. Finally, $\measuredangle XOY \cong \measuredangle X'OY'$, since they are opposite 'vertical angles'. Hence, $\Delta XOY \sim \Delta X'OY'$ and the top edge of the recto corresponds with the bottom edge of the two-page layout (mathematically, we write that $XY \leftrightarrow X'Y'$).

Not only is the top triangle XOY similar to the bottom triangle $X'OY'$, the top triangle is half the size as the bottom triangle. How do we know that? Because the width of the recto is half the width of the two-page spread. Therefore, every corresponding side of the top triangle is half the length of the corresponding side of the bottom triangle. Therefore, $XY = \frac{1}{2}X'Y'$, $XO = \frac{1}{2}X'O$ and $YO = \frac{1}{2}Y'O$.

In his 1983 article, Somerville posits the three major claims which form the basis of the golden canon. I will address and prove geometrically each one of these claims.

Claim 3.1: 'The text area always has the *same proportion* [Somerville's emphasis] as the page'.

To construct the text area, we choose an arbitrary point P on the line XO, and we draw a horizontal line until we intersect point A which is on line YO. As a result, we have formed $\Delta POA \sim \Delta XOY$. Line PA, which is the top of the text area, corresponds to line XY, which is the top of the page ($PA \leftrightarrow XY$). I should emphasize that the choice of the point P on the line XO can be completely arbitrary and the results hold, which is one of the beauties of this method.

Then we draw the vertical line until we intersect point B, which is on line $X'O$. So, we have formed $\Delta AOB \sim \Delta YOX'$. Line AB, which is the right side of the text area, corresponds to line YX', which is the right side of the page ($AB \leftrightarrow YX'$).

Therefore, the top of the text area has the same proportion as the top of the page and the right side of the text area has the same proportion as the right side of the page. Mathematically, we express this finding in the following manner:

$$\frac{PA}{XY} = \frac{AB}{YX'}$$

What about the bottom and the left side? Let us continue to form the text area.

From point B, we draw a horizontal line until it intersects with point C' on line $Y'O$. And from the starting point P, draw a vertical line until it intersects our new line BC'. That point of intersection is labelled C. So, now we have completed rectangle $PABC$.

To show equal proportionality of text area to page size on the bottom and left, we rely upon the principle that the diagonal of a rectangle forms two congruent right-angled triangles. The diagonal of rectangle $PABC$ is line PB, so that $\Delta PCB \cong \Delta BAP$. Notice that PA not only corresponds with BC ($PA \leftrightarrow BC$), but that they are congruent with each other ($PA \cong BC$). By the same token, PC not only corresponds with BA ($PC \leftrightarrow BA$), but they are also congruent with each other ($PC \cong BA$).

Meanwhile, the entire recto is itself a rectangle $XYX'W$. Line XX', of which line PB is a segment, forms the diagonal of rectangle $XYX'W$. Therefore, $\Delta XWX' \cong \Delta X'YX$. Once again, notice that XY not only corresponds with $X'W$ ($XY \leftrightarrow X'W$), but that they are congruent with each other ($XY \cong X'W$). By the same token, XW not only corresponds with $X'Y$ ($XW \leftrightarrow X'Y$), but they are congruent with each other ($XW \cong X'Y$).

The facts stated in the last two paragraphs allow us to set up the following ratios:

$$\frac{PA}{XY} = \frac{BC}{X'W}$$

Those same facts allow us to set up these ratios:

$$\frac{AB}{YX'} = \frac{CP}{WX}$$

So, now we can finally write

$$\frac{BC}{X'W} = \frac{CP}{WX}$$

Mathematically, the last equation states the following: line segment BC (the bottom of the recto's text area) is to line segment $X'W$ (the bottom of the recto itself) as line segment CP (the left side of the recto's text area) is to line segment WX (the left side of the recto itself).

We have proven Claim 3.1, but let us go one step further and prove a corollary:

Corollary 3.2: *If you draw a straight line from the bottom middle of the two-page layout (point W) to point O, you will always pass through the bottom left of the text area (point C).*

We have already proved that $\Delta POA \sim \Delta XOY$ and that $PA \leftrightarrow XY$. But $BC \leftrightarrow PA$ and $BC \cong PA$, while at the same time $X'W \leftrightarrow XY$ and $X'W \cong XY$. Furthermore, the two lines BC and $X'W$ both begin on the same side of the diagonal line XX'. So in the same way that $\Delta POA \sim \Delta XOY$ and $PA \leftrightarrow XY$, we have that $\Delta BOC \sim \Delta X'OW$ and that $BC \leftrightarrow X'W$. We could easily show that $\Delta COP \sim \Delta WOX$ and that $CP \leftrightarrow WX$.

Therefore, the line WO forms the boundary between triangle $X'OW$ and triangle WOX. On this line sits the point C, which proves the corollary.

Claim 3.3: 'The text area is always distributed within the page so that the outer margin is twice [the width of] the inner margin and the bottom margin is twice [the width of] the top margin'.

Since $\Delta XOY \sim \Delta X'OY'$, then all corresponding sides have the same proportion. And what is that proportion? It is $1/2$, because line XY (the width of the recto) is half the size of line $X'Y'$ (the width of the two-page spread).

Let us now construct a new vertical line DO. Starting from the top of the recto and moving straight down to point O, this line represents the height of triangle XOY. By the same token, we will also construct another new vertical line $D'O$. Starting from the bottom of the recto and moving straight up to point O, this line represents the height of triangle $X'OY'$. By similarity, we have the following proportion:

$$\frac{XY}{X'Y'} = \frac{XO}{X'O} = \frac{YO}{Y'O} = \frac{DO}{D'O} = \frac{1}{2}$$

We call the intersection of line PA with line DO point E, and in the same manner we call the intersection of line BC with line $D'O$ point E'.

In proving Claim 3.1, we have already shown that $PA \leftrightarrow BC$. Given that fact, then PA within triangle XOY corresponds to BC' within triangle $X'OY'$.

Now we can set up the following proportion:

$$\frac{DE}{D'E'} = \frac{1}{2}$$

Now we extend vertical line CP (the left side of the text area) until it intersects with the horizontal line forming the top of the page. We call the point of intersection V, so that PVX is a right angle. Therefore, we have a new right-angled triangle, PVX.

Next we extend vertical line AB (the right side of the text area) until it intersects with the horizontal line forming the bottom of the page. We call the point of intersection V', so that once again we have created a new right-angled triangle, $BV'X'$.

Moreover, $\Delta PVX \sim \Delta BV'X'$.

Finally observe these corresponding sides: $XV \leftrightarrow X'V'$ and $PV \leftrightarrow BV'$. Hence, we have these ratios:

$$\frac{XV}{X'V'} = \frac{PV}{BV'} = \frac{1}{2}$$

And that proves Claim 3.3 – that the text is set up so that the width of the outer margin ($X'V'$) is twice the width of the inner margin (XV) and the width of the bottom margin (BV') is twice the width of the top margin (PV).

Although we have completed the proof of Claim 3.3, let us prove one more corollary:

Corollary 3.4: *Point O is always one-third the distance from the top of the page to the bottom and two-thirds the distance from the bottom of the page to the top. Furthermore, point O is always one-third the distance from the left side of the page to the right side and two-thirds the distance from the right side of the page to the left side.*

To prove the first half of this corollary, we note that $XY = 1/2X'Y'$, so that $DO = 1/2D'O$ or $D'O = 2DO$.

To prove the second half takes just a bit longer, because for the first time we have to reference triangle XOZ. Clearly, $\Delta XOZ \sim \Delta X'OY$, but we also need to show that triangle XOZ is half the size of triangle $X'OY$. This is easy enough. Observe that, inside of the large right-angled triangle $X''YX'$, which occupies the top-right half of the two-page layout, we have the similar and smaller right-angled triangle $X''XZ$, which occupies the top right of the verso. Meanwhile, the width of the two-page spread is twice the width of the verso or mathematically we have that $X''X = 1/2X''Y$. Just as $X''X \leftrightarrow X''Y$, it also follows that $ZX \leftrightarrow X'Y$ and that $ZX = 1/2X'Y$. Therefore, $FO = 1/2F'O$ or $F'O = 2FO$.

That proves the corollary, and here is its significance. Even if the text area were reduced to the limit – to the single point O – then Claim 3.3 would still hold. That is, the text area would lie such that the outer margin (in this case, $F'O$) would be twice the width of the inner margin (FO) and the bottom margin ($D'O$) would be twice the width of the top margin (DO).

Claim 3.5: 'The ratios – inner margin to top margin, and outer margin to bottom margin – are always the same as the ratio of the width of the page to its height'.

To prove this, we observe the right-angled triangle occupying the upper right side of the recto, XYX', inside of and alongside of which are two additional right-angled triangles both similar to this larger triangle. We have $\Delta XVP \sim \Delta X'V'B \sim \Delta XYX'$.

We see that $XV \leftrightarrow XY \leftrightarrow X'V'$. This means that the inner margin is to the width of the page, as the width of the page is to the outer margin, and we also see that $PV \leftrightarrow X'Y \leftrightarrow BV'$. This means that the top margin is to the height of the page, as the height of the page is to the bottom margin.

We have proven Claim 3.5.

4. Conclusion

We have completed the geometrical proof. As far as I can ascertain, this article offers the first such proof that the text on a manuscript or a printed page can be placed according to the tenets of the golden canon, no matter the page proportions. Edwards gave the results, but not a proof. We began our drawing at an arbitrary point P on line OX. Therefore, the proof of the three claims, and of the two corollaries that I added, would always work no matter where we began drawing the text area on that line.

Even beyond the specifics of the geometry, the golden canon offers a fascinating example of how scribes beginning with the High Middle Ages, and printers and publishers of early modern Europe, may have applied mathematics to their profession in a surprisingly sophisticated manner. Whether they placed text on the page, deliberately using the method that Tschichold centuries later called the golden canon, merits further research. Moreover, if they did so, did they understand why the method worked? Again, this could be a topic of more study.

As a peripheral matter, I wanted to raise those questions. My principal objective in this article was to show why the golden canon does work by the use of geometry, however much area or however little area the text on a page occupies.

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