The ‘Golden Canon’ of book-page construction: proving the proportions geometrically

Stanley M. Max*

Department of Mathematics and Statistics, University of Southern Maine, Portland, ME 04104, USA

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Scholars of book arts have speculated upon and may have proven a deliberate method by which scribes and printers of past centuries placed the text area in manuscript and printed books on the page to render certain specific text-to-page ratios. According to this presumed method, the size and proportions of the text area would assume specific mathematical properties with respect to the size and proportions of the page. In 1953, one book-arts scholar, Jan Tschichold, referred to the ‘Golden Canon of Gothic book page construction’, and thus coined the term ‘golden canon’ (which, it should be noted, has nothing to do with the golden section). These scholars have stated that the golden canon results in these properties, but none of them have shown why it does. Using basic geometry, and in particular similar triangles, this article addresses the question.

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1. Background

Several scholars of book arts have speculated upon and may have proven a deliberate method by which scribes and printers of past centuries placed the text area in manuscript and printed books on the page to render certain specific text-to-page ratios. Especially notable among these scholars was Jan Tschichold [6], an eminent student of book design and typography. ‘After much toilsome work’, Tschichold wrote in 1975, ‘I finally succeeded, in 1953, in reconstructing the Golden Canon of book page construction as it was used during late Gothic times by the finest of scribes’. Tschichold – who had now coined the term ‘golden canon’ – went on to say that what he ‘uncovered as the canon of the manuscript writers, Raúl Rosarivo [another book-arts scholar] proved to have been Gutenberg’s canon as well’. In turn, Sam Somerville [5] synthesized the work of Tschichold, Rosarivo and other scholars in a noteworthy article published in 1983.

So then, what is the golden canon of book-page construction? It is a technique, Somerville explains, that places the text on a page with three predictable and consistent results. In my mind, the fact that the same text layout can always be reproduced yields both a certain aesthetic tranquillity as well as a scientific precision to the process of text composition. Shall we accept the claim of Rosarivo and Tschichold that medieval scribes and early modern printers were placing text on the page according to a certain set procedure that Tschichold called the golden canon?

That remains an open question and one inviting further investigation. If we do accept the claim, however, then the fact that such results were achieved by the High Middle Ages makes the student of European cultural history, and especially the student of book arts, marvel at the mathematical sophistication already achieved by that time. I think that the golden canon of book-page construction evokes the building skill of the Gothic architects who designed and constructed the magnificent European cathedrals. What those architects achieved in constructing a building, so too did scribes and printers accomplish in putting text onto a page.

To get down to brass tacks, just what are those three predictable and consistent results? First, the text area has the same proportion of width to height as the page does. Second, the width of the outer margin of the text area ends up being exactly twice the width of the inner margin, and the width of the bottom margin becomes exactly twice the width of the top margin. Third, the ratios of inner-margin width to top-margin width, and outer-margin width to bottom-margin width, turn out to be the very same ratio of page width to page height. (Incidentally, the reader should not confuse the golden canon with the golden section, which of course is the famous mathematical construction that also involves ratios but that does not necessarily have anything to do with page layout.)

Tschichold’s article showed how to construct a page using the golden canon so that it worked with
particular ratios of page width to page height, but he
did not explain why it worked with any page ratio. In
1919, Jay Hambidge [2,3] showed designs having what
he called ‘dynamic symmetry’, and Tschichold based
his article upon those designs. However, Hambidge
did not refer to specific text-to-page ratios. In 1932,
Edward B. Edwards [1] demonstrated how to lay out
the text on a page so that it yielded the proportions
that Tschichold years later called the golden canon, but
he did not address all three ratios emanating from the
golden canon. Other authors have also described page
layouts resembling the golden canon, but none of them
have fully explained the mathematical proportions
behind their layouts (see http://en.wikipedia.org/wiki/
Canons_of_page_construction and [4,7]). Somerville’s
synthesis clearly spelled out these three ratios produced
by the golden canon, which I have restated in the
paragraph above. Once more, however, Somerville did
not explain why those results occur.

My article will refer to Somerville’s synopsis of the
golden canon, and one by one I will explain why
the canon always produces specific ratios no matter the
size or proportions of the page. To do this, I will rely
upon the geometrical concept of similar triangles.

This article grew out of a summer programme in
book arts that I took at the University of Southern
Maine. One of the instructors in that programme,
Nancy Leavitt, a noted book artist and calligrapher,
introduced me to the subject of the golden canon, and
she gave me a copy of Somerville’s article. That article
ignited my fascination with this topic.

2. Similar triangles

Let us briefly review the concept of similar triangles.
Consider Figure 1, in which I have depicted two
triangles so that the smaller triangle on the left, \(ABC\), is
the same shape but half the linear size as the larger
triangle on the right, \(A'B'C'\). (The smaller triangle \(ABC\)
also occupies one-quarter the area of \(A'B'C'\), but that
fact does not concern us in this article.) Thus, the two
triangles are similar (which we write as \(\Delta ABC \sim \Delta A'B'C'\)). The two triangles are not congruent since
they have different sizes. If they were congruent, we
would write that \(\Delta ABC \cong \Delta A'B'C'\).

To explain necessary symbols, we observe that
angle \(ABC\) has the same measurement as angle \(A'B'C'\),
which geometers write as \(\angle ABC \equiv \angle A'B'C'\). In the
same manner, we have that \(\angle BAC \equiv \angle B'A'C'\) and
that \(\angle ACB \equiv \angle A'C'B'\). As a result of these equalities
of angle measurement, we have the situation that
side \(AC\) corresponds to side \(A'C'\) (geometers write that
\(AC \leftrightarrow A'C'\)), side \(CB\) corresponds to side \(C'B'\)
\((CB \leftrightarrow C'B')\) and side \(BA\) corresponds to side \(B'A'\)
\((BA \leftrightarrow B'A')\).

So what consequence do these correspondences
have? They mean that we have the following equal
proportion:

\[
\frac{AC}{A'C'} = \frac{CB}{C'B'} = \frac{BA}{B'A'}
\]

Furthermore, since I have deliberately drawn the
larger triangle to be twice the size of the smaller, so
as to simplify the arithmetic of the ratios, we can
quantify the ratios further. Using sides \(AC\) and \(A'C'\) as
an example (I could have used any one of the three
sides), we have the following equations:

\[
AC = \frac{1}{2} A'C' \text{ or } 2AC = A'C'
\]

The ability to establish and quantify proportions
has profound results for the golden canon, and indeed
it provides the ticket to prove the golden-canon ratios
by means of geometry.

3. Explanation of the diagrams

My article contains two figures besides the sample
triangles in Figure 1. Figure 2, commonly depicted in
books and articles on the subject, indicates how the
text areas are placed on the verso (left-hand page) and

![Figure 1](image1.png)

Figure 1. An example of two similar triangles. The smaller
triangle on the left, \(ABC\), is one-half the linear size of the
larger triangle on the right, \(A'B'C'\).

![Figure 2](image2.png)

Figure 2. The placement of the text area on the verso and the
recto according to the golden canon. The size of the text area
was arbitrary – any size would fulfil the properties of the
golden canon.
the recto (right-hand page) according to the golden canon. Somerville’s 1983 article contains a similar illustration along with annotations and labels of key lines and angles. Observe that Figure 2 depicts a two-page layout with the dotted line separating the pages. Two diagonals cross the two-page spread each going to opposite corners. The verso also has a diagonal proceeding from the bottom-left to the top-right corner and the recto has its own diagonal going from the top-left to the bottom-right corner. The two smaller rectangles, one on each page, indicate a possible area for text, and I will shortly explain how to construct those rectangles. The text area that I chose was arbitrary – as long as the text block is constructed in the manner that I am about to describe, the ratios predicted by the golden canon always work.

In Figure 3, I have continued to utilize Somerville’s labels to maintain consistency, but I have added numerous other labels to which I shall refer as I proceed with my argument. I have eliminated Somerville’s annotation along with most of the lines from the verso so that I can concentrate on the recto. Everything that I establish regarding the recto immediately applies to the verso, and therefore we have no need to clutter the diagram.

In Figure 3, to which the remainder of this article applies, I show two main diagonals. One of them extends from the bottom left of the two-page layout to the top right and the other one goes from the top left of the two-page layout to the bottom right. I also show a diagonal on the verso extending from its bottom left to its top right, as well as a diagonal on the recto going from its top left to its bottom right. I also have a dotted line bisecting the two-page layout.

We have that \( \triangle X'XY \cong \triangle XX'Y \), because they are congruent ‘alternate interior angles’. A straight line intersecting parallel lines forms alternate interior angles. Here, of course, the parallel lines are the top and bottom edges of the page and the straight line is \( XX' \). For the same reason, \( \triangle XYY' \cong \triangle X'Y'Y \). Finally, \( \triangle XOY \cong \triangle X'OY \), since they are opposite ‘vertical angles’. Hence, \( \angle XOY = \angle X'OY \) and the top edge of the recto corresponds with the bottom edge of the two-page layout (mathematically, we write that \( XY \leftrightarrow X'Y' \)).

Not only is the top triangle \( XOY \) similar to the bottom triangle \( X'Y' \), the top triangle is half the size as the bottom triangle. How do we know that? Because the width of the recto is half the width of the two-page spread. Therefore, every corresponding side of the top triangle is half the length of the corresponding side of the bottom triangle. Therefore, \( XY = \frac{1}{2}X'Y' \), \( XO = \frac{1}{2}X'O \) and \( YO = \frac{1}{2}YO \).

In his 1983 article, Somerville posits the three major claims which form the basis of the golden canon. I will address and prove geometrically each one of these claims.

**Claim 3.1:** ‘The text area always has the same proportion [Somerville’s emphasis] as the page’.

To construct the text area, we choose an arbitrary point \( P \) on the line \( XO \), and we draw a horizontal line until we intersect point \( A \) which is on line \( YO \). As a result, we have formed \( \triangle POA \sim \triangle XOY \). Line \( PA \), which is the top of the text area, corresponds to line \( XY \), which is the top of the page (\( PA \leftrightarrow XY \)). I should emphasize that the choice of the point \( P \) on the line \( XO \) can be completely arbitrary and the results hold, which is one of the beauties of this method.

Then we draw the vertical line until we intersect point \( B \), which is on line \( X'O \). So, we have formed \( \triangle AOB \sim \triangle YOX' \). Line \( AB \), which is the right side of the text area, corresponds to line \( YY' \), which is the right side of the page (\( AB \leftrightarrow YY' \)).

Therefore, the top of the text area has the same proportion as the top of the page and the right side of the text area has the same proportion as the right side of the page. Mathematically, we express this finding in the following manner:

\[
\frac{PA}{XY} = \frac{AB}{YY'}
\]

What about the bottom and the left side? Let us continue to form the text area.

From point \( B \), we draw a horizontal line until it intersects with point \( C \) on line \( Y'O \). And from the starting point \( P \), draw a vertical line until it intersects our new line \( BC' \). That point of intersection is labelled \( C \). So, now we have completed rectangle \( PABC \).
To show equal proportionality of text area to page size on the bottom and left, we rely upon the principle that the diagonal of a rectangle forms two congruent right-angled triangles. The diagonal of rectangle $PABC$ is line $PB$, so that $\triangle PCB \equiv \triangle BAP$. Notice that $PA$ not only corresponds with $BC$ ($PA \lll BC$), but that they are congruent with each other ($PA \equiv BC$). By the same token, $PC$ not only corresponds with $BA$ ($PC \lll BA$), but they are also congruent with each other ($PC \equiv BA$).

Meanwhile, the entire recto is itself a rectangle $XYXW$. Line $XX'$, of which line $PB$ is a segment, forms the diagonal of rectangle $XYXW$. Therefore, $\triangle XWX \equiv \triangle XYX$. Once again, notice that $XY$ not only corresponds with $X'W$ ($XY \lll X'W$), but that they are congruent with each other ($XY \equiv X'W$). By the same token, $WX$ not only corresponds with $XY$ ($XW \lll X'Y$), but they are congruent with each other ($WX \equiv X'Y$).

The facts stated in the last two paragraphs allow us to set up the following ratios:

$$\frac{PA}{XY} = \frac{BC}{X'W}$$

Those same facts allow us to set up these ratios:

$$\frac{AB}{YY'} = \frac{CP}{WX}$$

So, now we can finally write

$$\frac{BC}{X'W} = \frac{CP}{WX}$$

Mathematically, the last equation states the following: line segment $BC$ (the bottom of the recto’s text area) is to line segment $X'W$ (the bottom of the recto itself) as line segment $CP$ (the left side of the recto’s text area) is to line segment $WX$ (the left side of the recto itself).

We have proven Claim 3.1, but let us go one step further and prove a corollary:

**Corollary 3.2:** If you draw a straight line from the bottom middle of the two-page layout (point $W$) to point $O$, you will always pass through the bottom left of the text area (point $C$).

We have already proved that $\triangle POA \sim \triangle XOY$ and that $PA \lll XY$. But $BC \lll PA$ and $BC \equiv PA$, while at the same time $X'W \lll XY$ and $X'W \equiv XY$. Furthermore, the two lines $BC$ and $X'W$ both begin on the same side of the diagonal line $XY$. So in the same way that $\triangle POA \sim \triangle XOY$ and $PA \lll XY$, we have that $\triangle BOC \sim \triangle X'OW$ and that $BC \lll X'W$. We could easily show that $\triangle COP \sim \triangle WOX$ and that $CP \lll WX$.

Therefore, the line $WO$ forms the boundary between triangle $X'OW$ and triangle $WOX$. On this line sits the point $C$, which proves the corollary.

**Claim 3.3:** The text area is always distributed within the page so that the outer margin is twice [the width of] the inner margin and the bottom margin is twice [the width of] the top margin.

Since $\triangle XOY \sim \triangle X'OY'$, then all corresponding sides have the same proportion. And what is that proportion? It is $1/2$, because line $XY$ (the width of the recto) is half the size of line $X'Y'$ (the width of the two-page spread).

Let us now construct a new vertical line $DO$. Starting from the top of the recto and moving straight down to point $O$, this line represents the height of triangle $XOY$. By the same token, we will also construct another new vertical line $DO$. Starting from the bottom of the recto and moving straight up to point $O$, this line represents the height of triangle $X'OY'$. By similarity, we have the following proportion:

$$\frac{XY}{X'Y'} = \frac{XO}{X'O} = \frac{YO}{Y'O} = \frac{DO}{D'O} = \frac{1}{2}$$

We call the intersection of line $PA$ with line $DO$ point $E$, and in the same manner we call the intersection of line $BC$ with line $D'O$ point $E'$.

In proving Claim 3.1, we have already shown that $PA \lll BC$. Given that fact, then $PA$ within triangle $XOY$ corresponds to $BC$ within triangle $X'OY'$.

Now we can set up the following proportion:

$$\frac{DE}{D'E'} = \frac{1}{2}$$

Now we extend vertical line $CP$ (the left side of the text area) until it intersects with the horizontal line forming the top of the page. We call the point of intersection $V$, so that $PVX$ is a right angle. Therefore, we have a new right-angled triangle, $PVX$.

Next we extend vertical line $AB$ (the right side of the text area) until it intersects with the horizontal line forming the bottom of the page. We call the point of intersection $V'$, so that once again we have created a new right-angled triangle, $BV'X'$.

Moreover, $\triangle PVX \sim \triangle B'V'X'$. 

Finally observe these corresponding sides: $XV \lll X'V'$ and $PV \lll BV'$. Hence, we have these ratios:

$$\frac{XV}{X'V'} = \frac{PV}{BV'} = \frac{1}{2}$$

And that proves Claim 3.3 – that the text is set up so that the width of the outer margin ($X'V'$) is twice the width of the inner margin ($XV$) and the width of the bottom margin ($BV'$) is twice the width of the top margin ($PV$).
Although we have completed the proof of Claim 3.3, let us prove one more corollary:

**Corollary 3.4:** Point $O$ is always one-third the distance from the top of the page to the bottom and two-thirds the distance from the bottom of the page to the top. Furthermore, point $O$ is always one-third the distance from the left side of the page to the right side and two-thirds the distance from the right side of the page to the left side.

To prove the first half of this corollary, we note that $XY = 1/2XY'$, so that $DO = 1/2DO'$ or $DO = 2DO$.

To prove the second half takes just a bit longer, because for the first time we have to reference triangle $XOZ$. Clearly, $\Delta XOZ \sim \Delta XOY$, but we also need to show that triangle $XOZ$ is half the size of triangle $XOY$. This is easy enough. Observe that, inside of the large right-angled triangle $X'YX'$, which occupies the top-right half of the two-page layout, we have the similar and smaller right-angled triangle $X'ZX$, which occupies the top right of the verso. Meanwhile, the width of the two-page spread is twice the width of the verso or mathematically we have that $X''X = 1/2X''Y$. Just as $X''X \leftrightarrow X'Y$, it also follows that $ZX \leftrightarrow X'Y$ and that $ZX = 1/2XY$. Therefore, $FO = 1/2F'O$ or $F'O = 2FO$.

That proves the corollary, and here is its significance. Even if the text area were reduced to the limit - to the single point $O$ - then Claim 3.3 would still hold. That is, the text area would lie such that the outer margin (in this case, $F'O$) would be twice the width of the inner margin ($FO$) and the bottom margin ($DO'$) would be twice the width of the top margin ($DO$).

**Claim 3.5:** ‘The ratios – inner margin to top margin, and outer margin to bottom margin – are always the same as the ratio of the width of the page to its height’.

To prove this, we observe the right-angled triangle occupying the upper right side of the recto, $XYX'$, inside of and alongside of which are two additional right-angled triangles both similar to this larger triangle. We have $\Delta XVP \sim \Delta X'YP \sim \Delta XYY'$.

We see that $XY \leftrightarrow X'Y \leftrightarrow X''Y$. This means that the inner margin is to the width of the page, as the width of the page is to the outer margin, and we also see that $PV \leftrightarrow X'Y \leftrightarrow BV$. This means that the top margin is to the height of the page, as the height of the page is to the bottom margin.

We have proven Claim 3.5.

### 4. Conclusion

We have completed the geometrical proof. As far as I can ascertain, this article offers the first such proof that the text on a manuscript or a printed page can be placed according to the tenets of the golden canon, no matter the page proportions. Edwards gave the results, but not a proof. We began our drawing at an arbitrary point $P$ on line $OX$. Therefore, the proof of the three claims, and of the two corollaries that I added, would always work no matter where we began drawing the text area on that line.

Even beyond the specifics of the geometry, the golden canon offers a fascinating example of how scribes beginning with the High Middle Ages, and printers and publishers of early modern Europe, may have applied mathematics to their profession in a surprisingly sophisticated manner. Whether they placed text on the page, deliberately using the method that Tschichold centuries later called the golden canon, merits further research. Moreover, if they did so, did they understand why the method worked? Again, this could be a topic of more study.

As a peripheral matter, I wanted to raise those questions. My principal objective in this article was to show why the golden canon does work by the use of geometry, however much area or however little area the text on a page occupies.

### References


